

Part I: Multiple Choice

Complete each multiple choice item and place your answer on the Answer Sheet provided.
(15 marks)

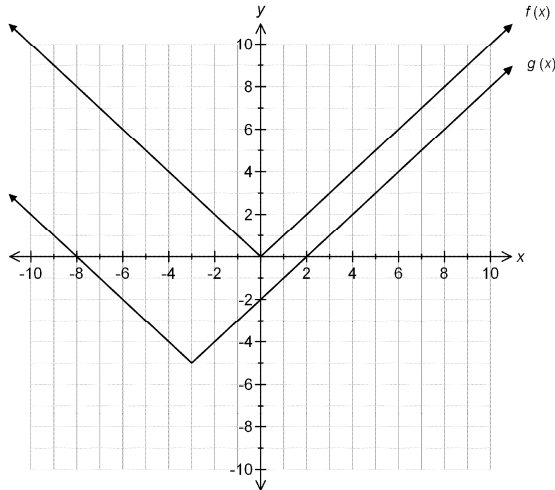
1. Which transformations occur when the vertex of the parabola $y = (x + 1)^2 - 5$ is mapped onto $(4, -7)$?

- A) Horizontal Translation of 3 units to the left
Vertical Translation of 2 units upward
- B) Horizontal Translation of 3 units to the right
Vertical Translation of 2 units downward
- C) Horizontal Translation of 5 units to the left
Vertical Translation of 2 units upward
- D) Horizontal Translation of 5 units to the right
Vertical Translation of 2 units downward

2. Which equation results when $y = f(x)$ is transformed using the mapping rule $(x, y) \rightarrow (x - 7, y + 1)$?

- A) $y = f(x - 7) - 1$
- B) $y = f(x - 7) + 1$
- C) $y = f(x + 7) - 1$
- D) $y = f(x + 7) + 1$

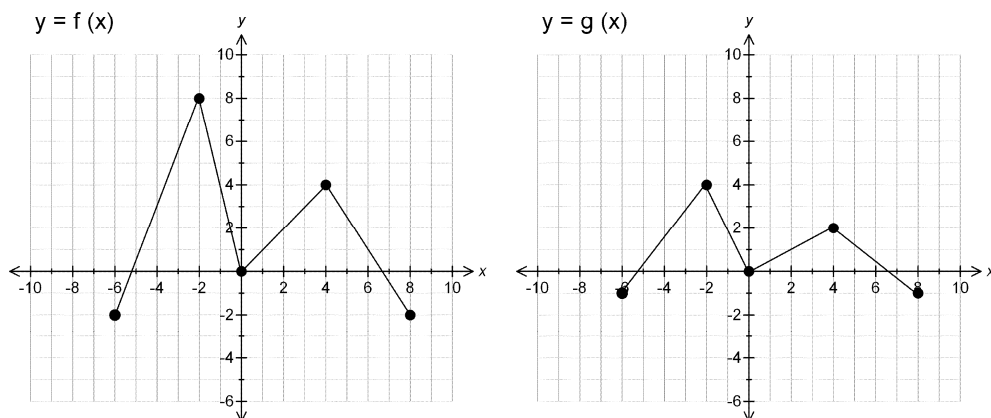
3. What is the equation of the transformed function $g(x)$ after the transformations are applied to the graph of $f(x) = |x|$?



- A) $g(x) + 3 = |x - 5|$
- B) $g(x) - 5 = |x + 3|$
- C) $g(x) = |x + 3| - 5$
- D) $g(x) = |x - 5| + 3$
4. Which statement is true when the graph of $y = f(x)$ is transformed onto $y = f\left(-\frac{1}{3}x + 6\right)$?
- A) Horizontal Stretch of $\frac{1}{3}$; Horizontal Translation 6 units to the left
- B) Horizontal Stretch of 3; Horizontal Translation 6 units to the right
- C) Horizontal Stretch of $\frac{1}{3}$; Horizontal Translation 18 units to the left
- D) Horizontal Stretch of 3; Horizontal Translation 18 units to the right

5. What are the coordinates of the invariant point(s) when the function $y = |x - 2|$ is reflected in the y-axis?
- A) $(-2, 0)$ and $(2, 0)$
- B) $(0, -2)$
- C) $(0, 2)$
- D) $(2, -2)$
6. Which equation represents the image of $y = (x - 2)^2$ after a reflection in the y-axis followed by a reflection in the x-axis?
- A) $y = (-x - 2)^2$
- B) $y = -(x - 2)^2$
- C) $y = -(x + 2)^2$
- D) $y = (-x + 2)^2$
7. What are the coordinates of the image point A' if point $A(-4, 6)$ on the function $f(x)$ is mapped onto the transformed function $g(x) = \frac{1}{3} f\left(\frac{1}{2}x\right)$?
- A) $(-8, 2)$
- B) $(-8, 18)$
- C) $(-2, 2)$
- D) $(-2, 18)$

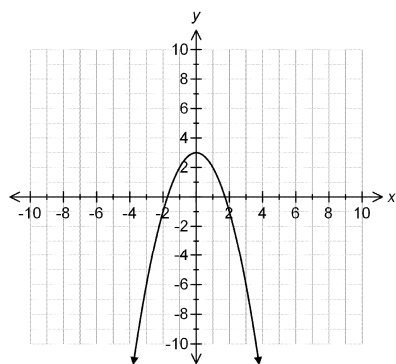
8. What transformation occurs when the graph of $y = f(x)$ is transformed onto the graph of $y = g(x)$?



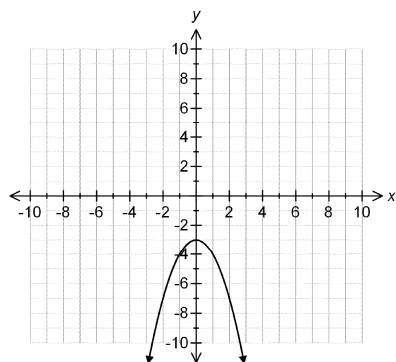
- A) vertical translation of 1 unit upward
- B) vertical translation of 1 unit downward
- C) vertical stretch by a factor of $\frac{1}{2}$
- D) vertical stretch by a factor of 2

9. Which graph results when $y = x^2$ is transformed onto $-y - 3 = (x)^2$?

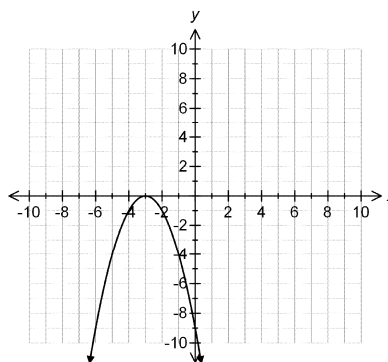
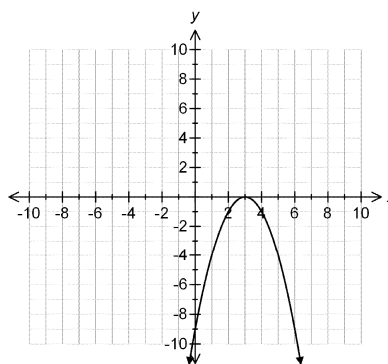
- A)
- C)



B)



D)

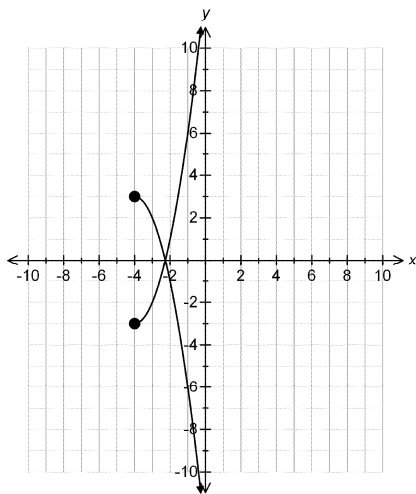


10. Which equation results when the function $f(x) = |x|$ is reflected in the x-axis, vertically stretched by the factor of $\frac{1}{3}$ and translated 4 units to the left?

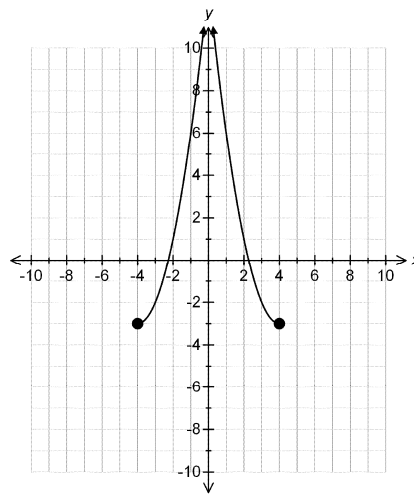
- A) $f(x) = -\frac{1}{3}|x+4|$
- B) $f(x) = -3|x+4|$
- C) $f(x) = -\frac{1}{3}|x| - 4$
- D) $f(x) = -3|x| - 4$

11. Which coordinate plane illustrates the graph of a function and its inverse?

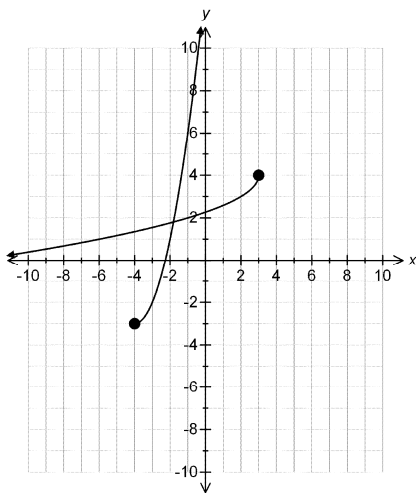
A)



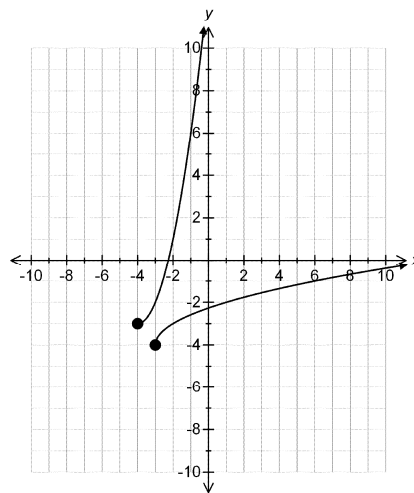
C)



B)



D)



12. Which mapping rule is used to sketch the graph of $y = f(x)$ and its inverse?

- A) $(x, y) \rightarrow \left(-x, \frac{1}{y}\right)$
- B) $(x, y) \rightarrow \left(x, -\frac{1}{y}\right)$
- C) $(x, y) \rightarrow (y, x)$
- D) $(x, y) \rightarrow (-y, -x)$

13. What is the equation of the inverse of $f(x) = \frac{1}{2}x - \frac{2}{3}$?

- A) $f^{-1}(x) = 2x + \frac{2}{3}$
- B) $f^{-1}(x) = 2x - \frac{3}{2}$
- C) $f^{-1}(x) = \frac{x+4}{3}$
- D) $f^{-1}(x) = \frac{6x+4}{3}$

14. What is the equation of $f^{-1}(x)$ if $f(x) = (x+3)^2$; $x \geq -3$?

- A) $f^{-1}(x) = \sqrt{x} - 3$; $x \geq 3$
- B) $f^{-1}(x) = \sqrt{x-3}$; $x \geq 3$
- C) $f^{-1}(x) = -3 + \sqrt{x}$; $x \geq 0$
- D) $f^{-1}(x) = -3 - \sqrt{x}$; $x \geq 0$

15. What is the domain of the inverse function of $f(x) = 3x^2 + 6x - 1$?

- A) $\{x \mid x \leq -1; x \in R\}$
- B) $\{x \mid x \geq -1; x \in R\}$
- C) $\{x \mid x \leq -4; x \in R\}$
- D) $\{x \mid x \geq -4; x \in R\}$

Name: _____

Part I: ANSWER SHEET

1. _____ 2. _____ 3. _____ 4. _____ 5. _____
6. _____ 7. _____ 8. _____ 9. _____ 10. _____
11. _____ 12. _____ 13. _____ 14. _____ 15. _____

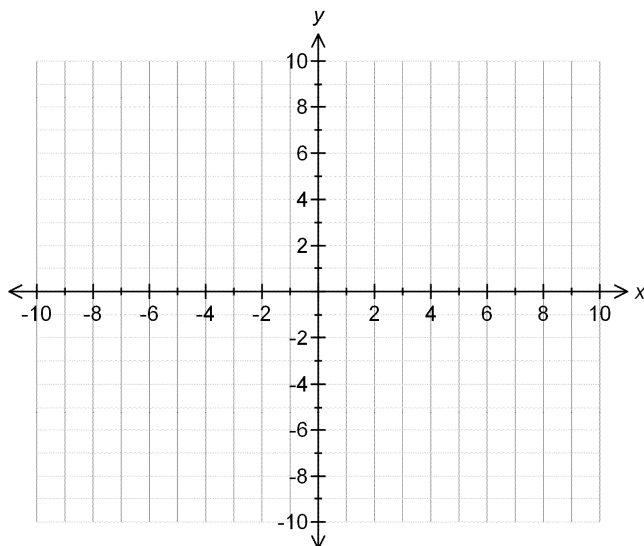
Part II: CONSTRUCTED RESPONSE

Complete each item in the space provided. Be sure to read each question carefully and provide all necessary details as part of your solution. (15 marks)

16. The graph of $f(x) = (x)^2$ is transformed to the graph of $g(x) = \frac{1}{2}(-x+4)^2 - 7$.

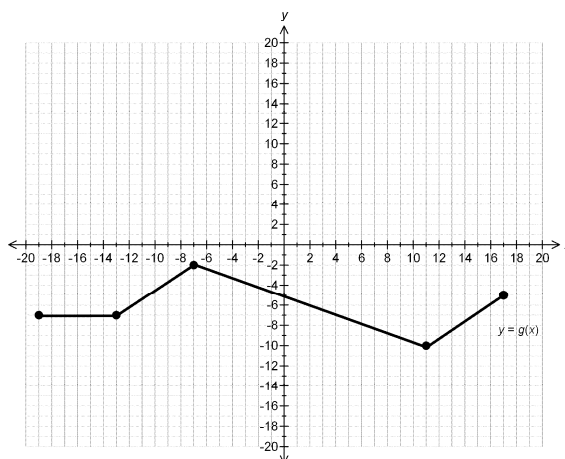
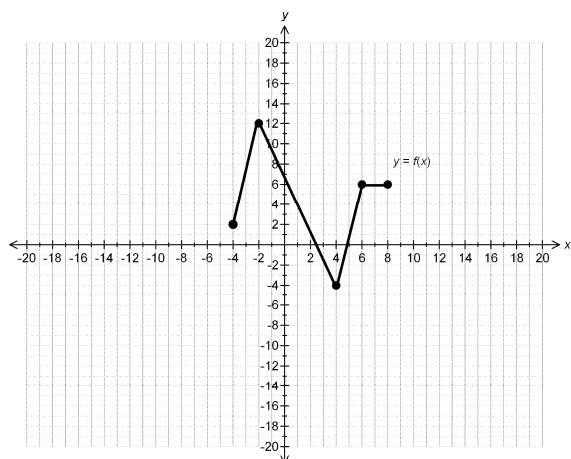
Use a mapping rule to sketch the graph of $g(x)$ and state the transformations that occur.

(4 marks)



17. The graph of $y = f(x)$ is transformed onto the graph of $y = g(x)$.

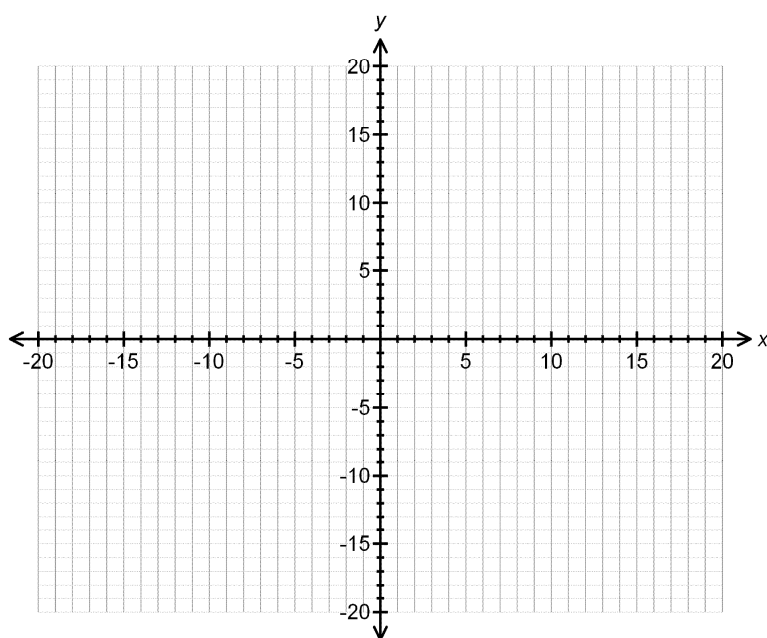
Describe the transformations that occur and algebraically determine the equation of $y = g(x)$ in the form $y = af(b(x - h)) + k$? (5 marks)



18. A) Restrict the domain of $f(x) = x^2 + 8x + 2$ so that the graph of its inverse is also a function.
(2 marks)

B) Algebraically determine the equation of $f^{-1}(x)$. (2 marks)

C) Sketch the graphs of $f(x)$ and $f^{-1}(x)$ on the same coordinate plane
and show the reflections in the line $y = x$. (2 marks)



Part I: Multiple Choice

Complete each multiple choice item and place your answer on the Answer Sheet provided.
(15 marks)

1. Which transformations occur when the vertex of the parabola $y = (x + 1)^2 - 5$ is mapped onto $(4, -7)$?

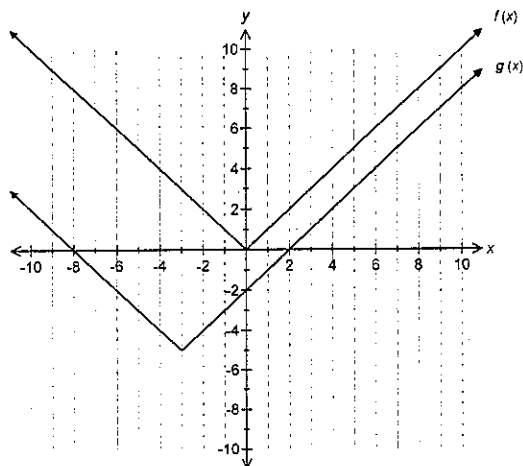
Vertex: $(-1, -5)$
 $(-1, -5) \rightarrow (4, -7)$
H.T. = 5 right
V.T. = -2 down

- A) Horizontal Translation of 3 units to the left
Vertical Translation of 2 units upward
- B) Horizontal Translation of 3 units to the right
Vertical Translation of 2 units downward
- C) Horizontal Translation of 5 units to the left
Vertical Translation of 2 units upward
- ☒ D) Horizontal Translation of 5 units to the right
Vertical Translation of 2 units downward
2. Which equation results when $y = f(x)$ is transformed using the mapping rule $(x, y) \rightarrow (x - 7, y + 1)$?

$$y - 1 = f(x + 7)$$
$$y = f(x + 7) + 1$$

- A) $y = f(x - 7) - 1$
- B) $y = f(x - 7) + 1$
- C) $y = f(x + 7) - 1$
- ☒ D) $y = f(x + 7) + 1$

3. What is the equation of the transformed function $g(x)$ after the transformations are applied to the graph of $f(x) = |x|$?



$$f(x) \rightarrow g(x)$$

$$(0,0) \rightarrow (-3,-5)$$

H.T. = -3 left V.T. = -5 down

$$\therefore g(x) + 5 = |x - (-3)|$$

$$g(x) + 5 = |x + 3|$$

$$g(x) = |x + 3| - 5$$

- A) $g(x) + 3 = |x - 5|$
- B) $g(x) - 5 = |x + 3|$
- ☒ C) $g(x) = |x + 3| - 5$
- D) $g(x) = |x - 5| + 3$
4. Which statement is true when the graph of $y = f(x)$ is transformed onto $y = f\left(-\frac{1}{3}x + 6\right)$?

A) Horizontal Stretch of $\frac{1}{3}$; Horizontal Translation 6 units to the left

B) Horizontal Stretch of 3; Horizontal Translation 6 units to the right

C) Horizontal Stretch of $\frac{1}{3}$; Horizontal Translation 18 units to the left

☒ D) Horizontal Stretch of 3; Horizontal Translation 18 units to the right

$$y = f\left(-\frac{1}{3}(x - 18)\right)$$

H.S. = 3 H.T. = 18 right

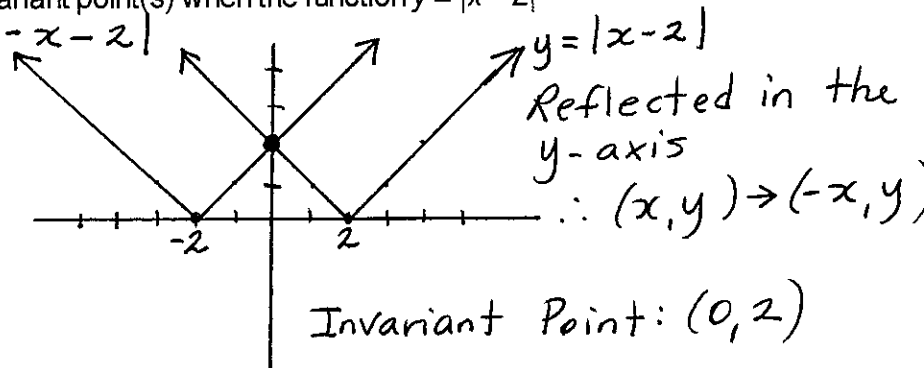
5. What are the coordinates of the invariant point(s) when the function $y = |x - 2|$ is reflected in the y-axis?

A) $(-2, 0)$ and $(2, 0)$

B) $(0, -2)$

☒ C) $(0, 2)$

D) $(2, -2)$



6. Which equation represents the image of $y = (x - 2)^2$ after a reflection in the y-axis followed by a reflection in the x-axis?

A) $y = (-x - 2)^2$

B) $y = -(x - 2)^2$

☒ C) $y = -(x + 2)^2$

D) $y = (-x + 2)^2$

Reflection in the y-axis: $(x, y) \rightarrow (-x, y)$
 $\therefore y = (-x - 2)^2$

Reflection in the x-axis: $(x, y) \rightarrow (x, -y)$
 $\therefore -y = (-x - 2)^2$
 $y = -(-(-x + 2))^2$
 $y = -(-1)^2(x + 2)^2 = -(x + 2)^2$

7. What are the coordinates of the image point A' if point $A(-4, 6)$ on the function $f(x)$

is mapped onto the transformed function $g(x) = \frac{1}{3}f\left(\frac{1}{2}x\right)$?

☒ A) $(-8, 2)$

B) $(-8, 18)$

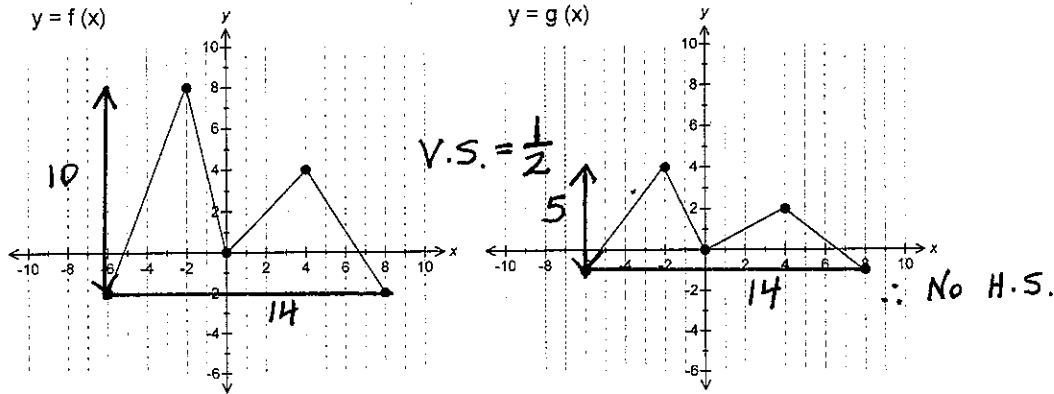
C) $(-2, 2)$

D) $(-2, 18)$

$3g(x) = f\left(\frac{1}{2}x\right)$
 \uparrow V.S. = $\frac{1}{3}$ \uparrow H.S. = 2

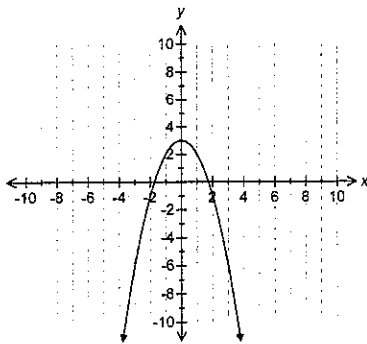
Mapping Rule
 $(x, y) \rightarrow (2x, \frac{1}{3}y)$
 $A(-4, 6) \rightarrow (-8, 2)$

8. What transformation occurs when the graph of $y = f(x)$ is transformed onto the graph of $y = g(x)$?

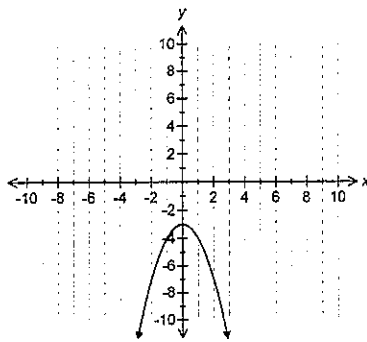


- A) vertical translation of 1 unit upward
B) vertical translation of 1 unit downward
C) vertical stretch by a factor of $\frac{1}{2}$
D) vertical stretch by a factor of 2
9. Which graph results when $y = x^2$ is transformed onto $-y - 3 = (x)^2$?

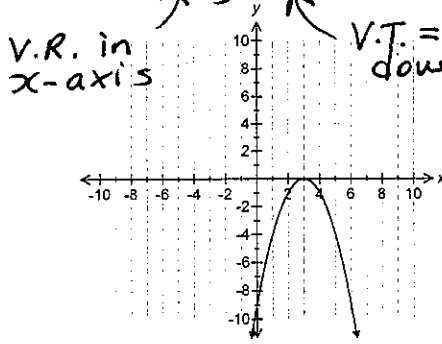
A)



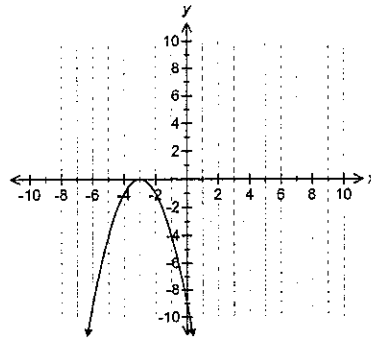
B)



C)



D)



$-y - 3 = (x)^2$
 $\uparrow (y + 3) = (x)^2$
 V.R. in x-axis
 V.T. = -3 down
 vertex $(0, -3)$
 Mapping Rule
 $(x, y) \rightarrow (x, -y - 3)$
 $(0, 0) \rightarrow (0, -3)$

10. Which equation results when the function $f(x) = |x|$ is reflected in the x-axis,

vertically stretched by the factor of $\frac{1}{3}$ and translated 4 units to the left?

(A) $f(x) = -\frac{1}{3}|x+4|$

B) $f(x) = -3|x+4|$

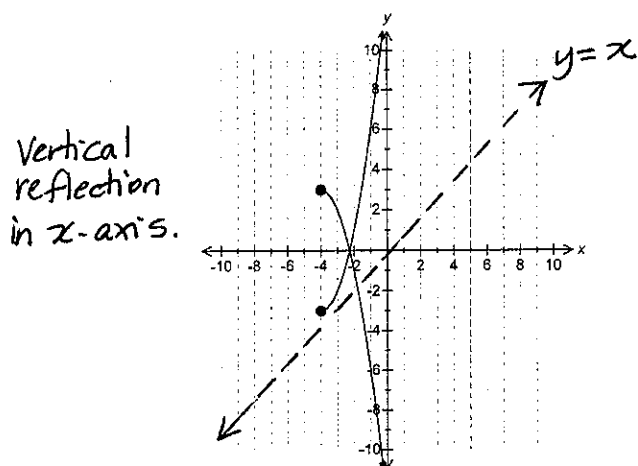
C) $f(x) = -\frac{1}{3}|x| - 4$

D) $f(x) = -3|x| - 4$

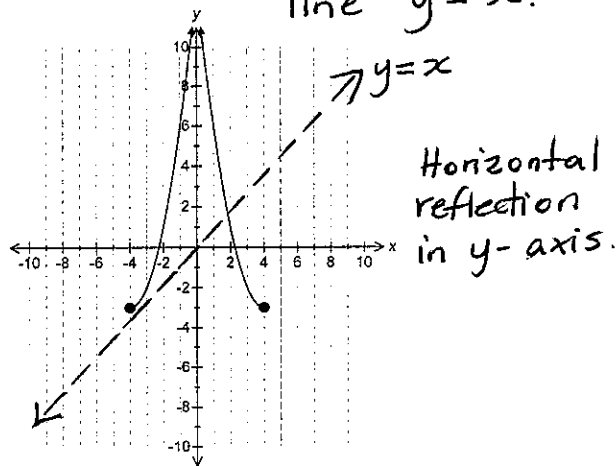
Reflected in x-axis : (V.R) $\therefore -f(x) = |x|$
V.S. = $\frac{1}{3}$ $\therefore -\frac{1}{3}f(x) = |x|$
H.T. = -4 (left) $\therefore -\frac{1}{3}f(x) = |x - (-4)|$
 $f(x) = -\frac{1}{3}|x+4|$

11. Which coordinate plane illustrates the graph of a function and its inverse? - reflections of each other in the line $y = x$.

A)

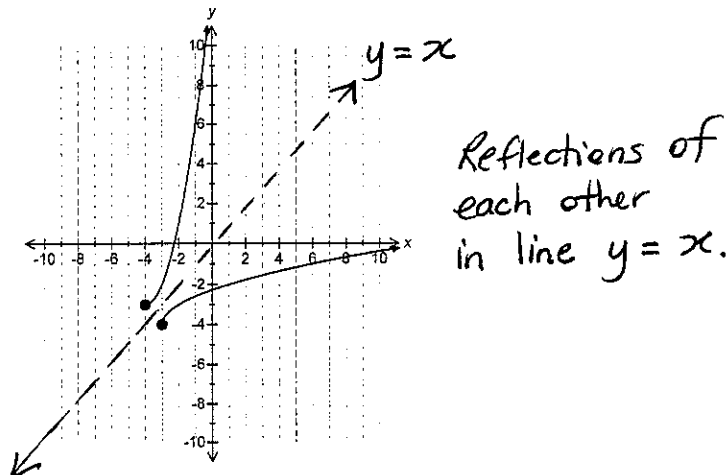
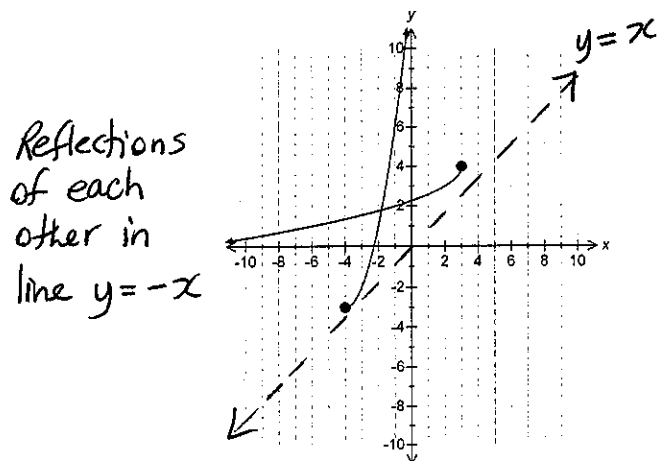


C)



B)

(D)



12. Which mapping rule is used to sketch the graph of $y = f(x)$ and its inverse?

- A) $(x, y) \rightarrow \left(-x, \frac{1}{y}\right)$
 B) $(x, y) \rightarrow \left(x, -\frac{1}{y}\right)$
 C) $(x, y) \rightarrow (y, x)$
 D) $(x, y) \rightarrow (-y, -x)$

switch x and y
 $\therefore (x, y) \rightarrow (y, x)$

13. What is the equation of the inverse of $f(x) = \frac{1}{2}x - \frac{2}{3}$?

- A) $f^{-1}(x) = 2x + \frac{2}{3}$
 B) $f^{-1}(x) = 2x - \frac{3}{2}$
 C) $f^{-1}(x) = \frac{x+4}{3}$
 D) $f^{-1}(x) = \frac{6x+4}{3}$

switch x and y
 solve for y
 $\therefore x = \frac{1}{2}y - \frac{2}{3}$
 $\therefore \frac{1}{2}y = x + \frac{2}{3} \quad (\times 2)$
 $y = 2\left(x + \frac{2}{3}\right)$
 $y = 2x + \frac{4}{3} = \frac{6x}{3} + \frac{4}{3}$
 $\therefore f^{-1}(x) = \frac{6x+4}{3}$

14. What is the equation of $f^{-1}(x)$ if $f(x) = (x+3)^2$; $x \geq -3$?

- A) $f^{-1}(x) = \sqrt{x} - 3$; $x \geq 3$
 B) $f^{-1}(x) = \sqrt{x-3}$; $x \geq 3$
 C) $f^{-1}(x) = -3 + \sqrt{x}$; $x \geq 0$
 D) $f^{-1}(x) = -3 - \sqrt{x}$; $x \geq 0$

$x = (y+3)^2$
 $\sqrt{(y+3)^2} = \pm \sqrt{x}$
 $y+3 = \pm \sqrt{x}$
 $y = -3 \pm \sqrt{x}$
 $\therefore f^{-1}(x) = -3 + \sqrt{x}$; $x \geq 0$

15. What is the domain of the inverse function of $f(x) = 3x^2 + 6x - 1$?

- A) $\{x \mid x \leq -1; x \in \mathbb{R}\}$
 B) $\{x \mid x \geq -1; x \in \mathbb{R}\}$
 C) $\{x \mid x \leq -4; x \in \mathbb{R}\}$
 D) $\{x \mid x \geq -4; x \in \mathbb{R}\}$

vertex: $x = \frac{-b}{2a} = \frac{-6}{2(3)} = -1$
 $y = 3(-1)^2 + 6(-1) - 1 = -4$
 $(-1, -4)$
 parabola opens upward

$D_{f(x)} : \{x \mid x \geq -1, x \in \mathbb{R}\} \rightarrow R_{f^{-1}(x)}$
 Restriction

$R_{f(x)} : \{y \mid y \geq -4, y \in \mathbb{R}\} \rightarrow D_{f^{-1}(x)}$

$\therefore D_{f^{-1}(x)} : \{x \mid x \geq -4, x \in \mathbb{R}\}$

Name: Solutions

Part I: ANSWER SHEET

1. D 2. D 3. C 4. D 5. C
6. C 7. A 8. C 9. B 10. A
11. D 12. C 13. D 14. C 15. D

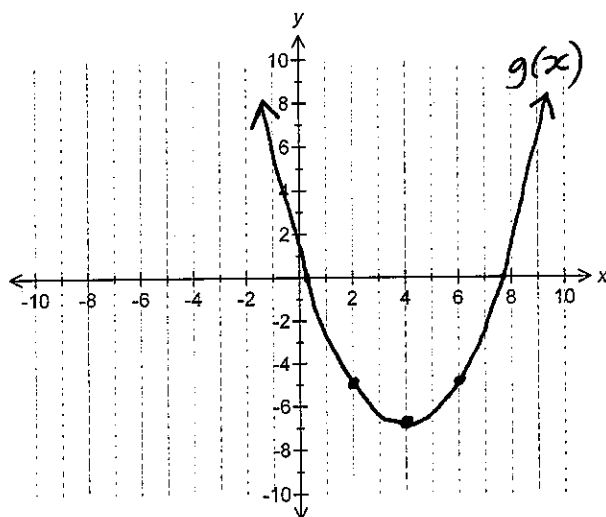
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Complete each item in the space provided. Be sure to read each question carefully and provide all necessary details as part of your solution. (15 marks)

16. The graph of $f(x) = (x)^2$ is transformed to the graph of $g(x) = \frac{1}{2}(-x+4)^2 - 7$.

Use a mapping rule to sketch the graph of $g(x)$ and state the transformations that occur.

(4 marks)



$$g(x) + 7 = \frac{1}{2}(-x+4)^2$$

$$2(g(x) + 7) = (-x+4)^2$$

\uparrow V.S. = $\frac{1}{2}$ \uparrow V.T. = -7 down \uparrow H.R. in y-axis \uparrow H.T. = 4 right

Mapping Rule:

$$(x, y) \rightarrow (-x+4, \frac{1}{2}y-7)$$

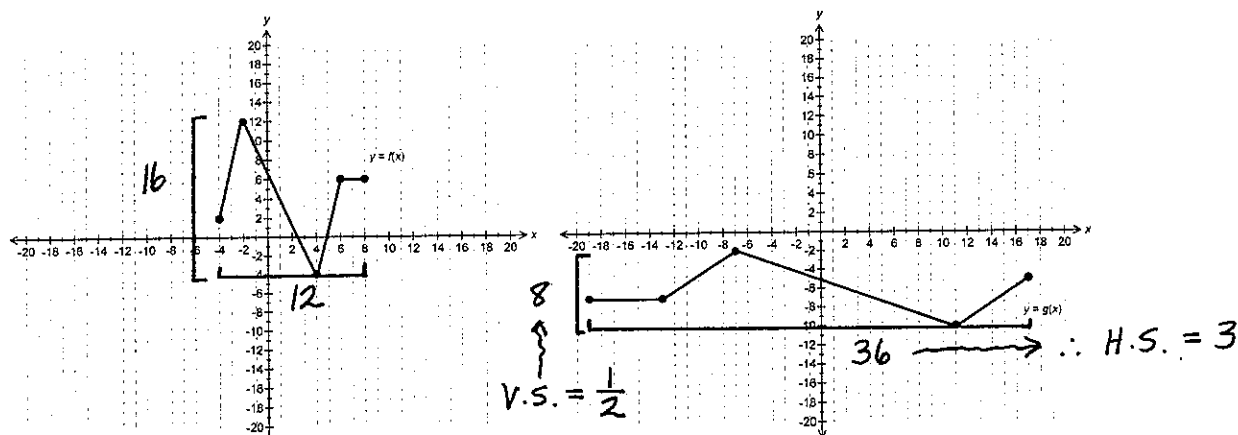
$$(-2, 4) \rightarrow (6, -5)$$

$$(0, 0) \rightarrow (4, -7)$$

$$(2, 4) \rightarrow (2, -5)$$

17. The graph of $y = f(x)$ is transformed onto the graph of $y = g(x)$.

Describe the transformations that occur and algebraically determine the equation of $y = g(x)$ in the form $y = af(b(x-h)) + k$? (5 marks) OR $\frac{1}{a}(y-k) = f(b(x-h))$



Reflections: Horizontal Reflection in y-axis $\therefore b < 0$
Vertical Reflection in x-axis $\therefore \frac{1}{a} < 0$

Stretches: H.S. = 3 $\therefore b = \frac{1}{3}$
V.S. = $\frac{1}{2}$ $\therefore \frac{1}{a} = 2$ so $a = \frac{1}{2}$

Translations: $f(x) \rightarrow g(x)$
 $(8, 6) \rightarrow (-19, -6)$
H.R. $(x, y) \rightarrow (-x, y)$ $\therefore (8, 6) \rightarrow (-8, 6)$
V.R. $(x, y) \rightarrow (x, -y)$ $\therefore (-8, 6) \rightarrow (-8, -6)$
H.S. = 3 $(x, y) \rightarrow (3x, y)$ $\therefore (-8, -6) \rightarrow (-24, -6)$
V.S. = $\frac{1}{2}$ $(x, y) \rightarrow (x, \frac{1}{2}y)$ $\therefore (-24, -6) \rightarrow (-24, -3)$

$(-24, -3) \rightarrow (-19, -6)$
H.T. = 5 right $\therefore h = 5$
V.T. = -3 down $\therefore k = -3$

Equation: $g(x) = -\frac{1}{2} f\left(-\frac{1}{3}(x-5)\right) - 3$

18. A) Restrict the domain of $f(x) = x^2 + 8x + 2$ so that the graph of its inverse is also a function.

(2 marks)

parabola opens upward
vertex: $x = \frac{-b}{2a} = \frac{-8}{2(1)} = -4$
 $y = (-4)^2 + 8(-4) + 2 = -14$
 $(-4, -14)$

Restriction:

$\{x \mid x \geq -4, x \in \mathbb{R}\}$

- B) Algebraically determine the equation of $f^{-1}(x)$. (2 marks)

switch x and y : $x = y^2 + 8y + 2$

solve for y : $y^2 + 8y = x - 2$

$y^2 + 8y + 16 = x - 2 + 16$

$(y + 4)(y + 4) = x + 14$

$(y + 4)^2 = x + 14$

$\rightarrow \sqrt{(y + 4)^2} = \pm \sqrt{x + 14}$

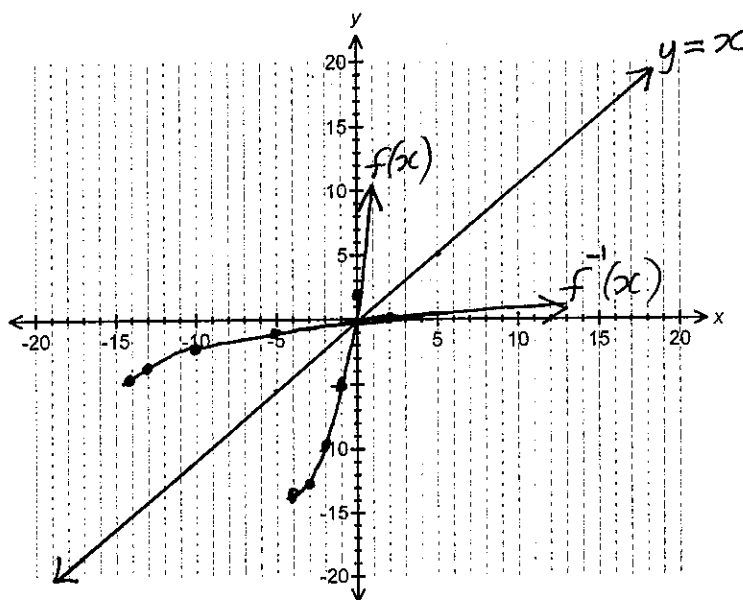
$y + 4 = \pm \sqrt{x + 14}$

$y = -4 \pm \sqrt{x + 14}$

$\therefore f^{-1}(x) = -4 + \sqrt{x + 14}; x \geq -14$

- C) Sketch the graphs of $f(x)$ and $f^{-1}(x)$ on the same coordinate plane

and show the reflections in the line $y = x$. (2 marks)



$f(x)$	
x	y
-4	-14
-3	-13
-2	-10
-1	-5
0	2

$f^{-1}(x)$	
x	y
-14	-4
-13	-3
-10	-2
-5	-1
2	0